1) $\int \; dx$ = x+c

2) $\int \; \propto dx = \propto \int \; dx = \propto$ (x + c)

3) $\int \; (\propto f(x) \; + \int \; \beta g(x))dx \; = \propto \int \; f(x) \; dx \; + \beta \int \; g(x) \; dx$

where $\propto, \beta$ are constant

4) $\int \; (f(x))^n \; . \, {}^{/}_{f}(x)dx \; = \frac{f(x))^{n+1}}{n+1}$ +c where n $\neq$ -1

Example : Evaluate the following integration

1- $\int \; 2x \; (x^2+1)^{23} \; dx$

$= \int \; (x^2+1)^{23} \, .2x \; dx = \frac{(x^2+1)^{24}}{24}$ +c

$2- \int \; y \, \sqrt{1+4y^2}$ dy

$= \int \; (1+4y^2)^{\frac{1}{2}} \; .ydy = \frac{1}{8} \int \; (1+4y^2)^{\frac{1}{2}} \; .8ydy = \frac{1}{8} \; \frac{(1+4y^2)^{\frac{3}{2}}}{\frac{3}{2}}$ + c

Theorem (1): If a and b are any positive number then :

1) $\ln(\mathrm{a.b})$ = ln(a) + ln(b)
2) ln(a/b) = ln(a) - ln(b)
3) ln(1) = 0
4) $\ln(a^r)$ = r $\ln(a)$ where r is rational number .

Theorem (2): If f(x)= $\ln(x)$ then

$\frac{d}{dx}(\ln|x|) = \frac{1}{u} \cdot \frac{du}{dx}$ if u > 0

$= \frac{-1}{u} \cdot \frac{d(-u)}{dx}$ if u < 0

$= \frac{1}{u} \cdot \frac{du}{dx}$

Example: Find the $\frac{dt}{dx}$ of the following

1-Dy = $\ln(x^3-3x+1)$

$\frac{dy}{dx} = \frac{1}{x^3-3x+1} \cdot \; 3x^2-3 = \frac{3x^2-3}{x^3-3x+1}$

2) y = $\ln(\sec(x)+\tan(x))$

$\frac{dy}{dx} = \frac{1}{\sec(x)+\tan(x)} \cdot \; \sec(x)\tan(x)+\sec(x)^2$

$= \frac{\sec(x)\tan(x)+\sec(x)^2}{\sec(x)+\tan(x)} = \sec(x)$

3-) ln y = $\ln(\frac{(x+1)(2x+3)^{\frac{2}{3}}}{x^2+3x+1})$ find $\frac{dy}{dx}$

ln y = $\ln(x+1)(2x+3)^{\frac{2}{3}}$ - $\ln(x^2+3x+1)$

ln y = $\ln(x+1)+\frac{2}{3}\ln(2x+3)$ - $\ln(x^2+3x+1)$

$\frac{1}{y}.\frac{dy}{dx} = \frac{1}{(x+1)} + \frac{2}{3}\frac{2}{(2x+3)} - \frac{1}{(2x^2+3x+1)}.2x \quad +3$

$\frac{1}{y}.\frac{dy}{dx} = \frac{1}{(x+1)} + \frac{4}{6x+9)} - \frac{2x \quad +3}{(2x^2+3x+1)}$ multiply by (y) both side And compensation in(y) value we get

$\frac{dy}{dx} = (\frac{(x+1)(2x+3)^{\frac{2}{3}}}{x^2+3x+1}) .[\frac{1}{(x+1)} + \frac{4}{6x+9)} - \frac{2x \quad +3}{(2x^2+3x+1)}]$

$000000000000000000000000000000000000000$

- $y = \sqrt{x(x+1)} = \sqrt{x}\ .\sqrt{x+1}$

y = $x^{\frac{1}{2}}.(x+1)^{\frac{1}{2}}$ by take the ln for both side we get

y = $\ln(x^{\frac{1}{2}}.(x+1)^{\frac{1}{2}})$

ln y= $\frac{1}{2}$ lnx + $\frac{1}{2}$ ln(x + 1) by derivative both side

$\frac{1}{y}.\frac{dy}{dx} = \frac{1}{2}.\frac{1}{x} + \frac{1}{2}\frac{1}{(x+1)}$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}(.\frac{1}{x} + \frac{1}{(x+1)})$

$\frac{1}{y}.\frac{dy}{dx} = (\frac{1}{2}(\frac{(x+1)+x}{x(x+1)}))$ multiply in y value from above

$= y(\frac{1}{2}(\frac{(x+1)+x}{x(x+1)}))$

$= \frac{1}{2}\sqrt{x(x+1)} \quad (\frac{2x+1}{x(x+1)}) = \frac{2x+1}{2x(x+1)}$

* Integral leading to $\ln(u)$

1) $\int \frac{1}{u} \, du = \ln u + c$.

- If the Derivative of the Denominator is equal to the Numerator, then the integration equals the value of the denominator.
-

Example :

Evaluate the following integration :

2) $\int \frac{\sin(x)}{\cos(x)} \, dx = -\ln \cos(x) + c$.

3) $\int \cot(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx = -\ln \sin(x) + c$

4) $\int \text{ssec}(x) \, dx$ H.W

5) $\int \text{csec}(x) \, dx$ H.W